# Outline of trig integration rules

1. $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Sin}[n (c + d x)])^p \, dx$ when $p \in \mathbb{Z}^- \land m \in \mathbb{Z} \land n \in \mathbb{Z}$

    1: $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Sin}[n (c + d x)])^p \, dx$ when $p \in \mathbb{Z}^- \bigwedge \frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n}{2} \in \mathbb{Z}$

    2. $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Sin}[n (c + d x)])^p \, dx$ when $p \in \mathbb{Z}^- \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$

        1: $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Sin}[n (c + d x)])^p \, dx$ when $\frac{p}{2} \in \mathbb{Z}^- \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$

        2: $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Sin}[n (c + d x)])^p \, dx$ when $\frac{p-1}{2} \in \mathbb{Z}^- \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$

    3: $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Sin}[n (c + d x)])^p \, dx$ when $p \in \mathbb{Z}^- \bigwedge \frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$

2. $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Cos}[n (c + d x)])^p \, dx$ when $p \in \mathbb{Z}^- \land m \in \mathbb{Z} \land n \in \mathbb{Z}$

    1: $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Cos}[n (c + d x)])^p \, dx$ when $p \in \mathbb{Z}^- \bigwedge \frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n}{2} \in \mathbb{Z}$

    2: $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Cos}[n (c + d x)])^p \, dx$ when $\frac{p-1}{2} \in \mathbb{Z}^- \bigwedge \frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$

    3: $\int (a \operatorname{Sin}[m (c + d x)] + b \operatorname{Cos}[n (c + d x)])^p \, dx$ when $p \in \mathbb{Z}^- \land m \in \mathbb{Z} \land n \in \mathbb{Z}$

# Rules for integrands of the form $(a \operatorname{Sin}[m (c+d x)]+b \operatorname{Sin}[n (c+d x)])^p$

1: $\int (a \operatorname{Sin}[m (c+d x)]+b \operatorname{Sin}[n (c+d x)])^p \, dx$ when $p \in \mathbb{Z}^- \bigwedge \frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n}{2} \in \mathbb{Z}$

- **Derivation: Integration by substitution**
- **Basis: If $\frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n}{2} \in \mathbb{Z}$, then**
  $F[\operatorname{Sin}[m (c+d x)], \operatorname{Sin}[n (c+d x)]] == \frac{1}{d} \operatorname{Subst}\left[\frac{F[\operatorname{Sin}[m \operatorname{ArcTan}[x]], \operatorname{Sin}[n \operatorname{ArcTan}[x]]]}{1+x^2}, x, \operatorname{Tan}[c+d x]\right] \partial_x \operatorname{Tan}[c+d x]$
- **Basis: If $\frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n}{2} \in \mathbb{Z}$, then**
  $F[\operatorname{Cos}[m (c+d x)], \operatorname{Cos}[n (c+d x)]] == -\frac{1}{d} \operatorname{Subst}\left[\frac{F[\operatorname{Cos}[m \operatorname{ArcCot}[x]], \operatorname{Cos}[n \operatorname{ArcCot}[x]]]}{1+x^2}, x, \operatorname{Cot}[c+d x]\right] \partial_x \operatorname{Cot}[c+d x]$
- **Note: If $\frac{m}{2} \in \mathbb{Z}$, then $\operatorname{Sin}[m \operatorname{ArcTan}[x]]$ equals $x$ times a rational functions in $x^2$.**
- **Note: If $\frac{m}{2} \in \mathbb{Z}$, then $\operatorname{Cos}[m \operatorname{ArcCot}[x]]$ equals a rational functions in $x^2$.**
- **Rule: If $p \in \mathbb{Z}^- \bigwedge \frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n}{2} \in \mathbb{Z}$, then**

$$\int (a \operatorname{Sin}[m (c+d x)]+b \operatorname{Sin}[n (c+d x)])^p \, dx \longrightarrow \frac{1}{d} \operatorname{Subst}\left[\int \frac{(a \operatorname{Sin}[m \operatorname{ArcTan}[x]]+b \operatorname{Sin}[n \operatorname{ArcTan}[x]])^p}{1+x^2} \, dx, x, \operatorname{Tan}[c+d x]\right]$$

- **Program code:**

```
Int[(a_.*sin[m_.*(c_.+d_.*x_)]+b_.*sin[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  1/d * Subst[Int[Simplify[TrigExpand[a*Sin[m*ArcTan[x]]+b*Sin[n*ArcTan[x]]]]^p/(1+x^2),x],x,Tan[c+d*x]] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && IntegerQ[m/2] && IntegerQ[n/2]
```

```
Int[(a_.*cos[m_.*(c_.+d_.*x_)]+b_.*cos[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  -1/d * Subst[Int[Simplify[TrigExpand[a*Cos[m*ArcCot[x]]+b*Cos[n*ArcCot[x]]]]^p/(1+x^2),x],x,Cot[c+d*x]] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && IntegerQ[m/2] && IntegerQ[n/2]
```

2. $\int (a \sin[m (c+d x)] + b \sin[n (c+d x)])^p \, dx$ when $p \in \mathbb{Z}^- \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$

1: $\int (a \sin[m (c+d x)] + b \sin[n (c+d x)])^p \, dx$ when $\frac{p}{2} \in \mathbb{Z}^- \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$

- **Derivation: Integration by substitution**
- **Basis: If** $\frac{p}{2} \in \mathbb{Z} \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$**, then**
  $(a \sin[m (c+d x)] + b \sin[n (c+d x)])^p = \frac{1}{d} \text{Subst}\left[\frac{(a \sin[m \text{ArcTan}[x]] + b \sin[n \text{ArcTan}[x]])^p}{1+x^2}, x, \tan[c+d x]\right] \partial_x \tan[c+d x]$
- **Basis: If** $\frac{p}{2} \in \mathbb{Z} \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$**, then**
  $(a \cos[m (c+d x)] + b \cos[n (c+d x)])^p = \frac{1}{d} \text{Subst}\left[\frac{(a \cos[m \text{ArcTan}[x]] + b \cos[n \text{ArcTan}[x]])^p}{1+x^2}, x, \tan[c+d x]\right] \partial_x \tan[c+d x]$
- **Note: If** $\frac{m-1}{2} \in \mathbb{Z}$**, then** $\sin[m \text{ArcTan}[x]]$ **equals** $\frac{x}{\sqrt{1+x^2}}$ **times a rational functions in** $x^2$**.**
- **Note: If** $\frac{m-1}{2} \in \mathbb{Z}$**, then** $\cos[m \text{ArcTan}[x]]$ **equals** $\frac{1}{\sqrt{1+x^2}}$ **times a rational functions in** $x^2$**.**
- **Rule: If** $\frac{p}{2} \in \mathbb{Z}^- \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$**, then**

$$\int (a \sin[m (c+d x)] + b \sin[n (c+d x)])^p \, dx \longrightarrow \frac{1}{d} \text{Subst}\left[\int \frac{(a \sin[m \text{ArcTan}[x]] + b \sin[n \text{ArcTan}[x]])^p}{1+x^2} \, dx, x, \tan[c+d x]\right]$$

$$\int (a \cos[m (c+d x)] + b \cos[n (c+d x)])^p \, dx \longrightarrow \frac{1}{d} \text{Subst}\left[\int \frac{(a \cos[m \text{ArcTan}[x]] + b \cos[n \text{ArcTan}[x]])^p}{1+x^2} \, dx, x, \tan[c+d x]\right]$$

- **Program code:**

```
Int[(a_.*sin[m_.*(c_.+d_.*x_)]+b_.*sin[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  1/d * Subst[Int[Simplify[TrigExpand[a*Sin[m*ArcTan[x]]+b*Sin[n*ArcTan[x]]]]^p/(1+x^2),x],x,Tan[c+d*x]] /;
FreeQ[{a,b,c,d},x] && ILtQ[p/2,0] && IntegerQ[(m-1)/2] && IntegerQ[(n-1)/2]
```

```
Int[(a_.*cos[m_.*(c_.+d_.*x_)]+b_.*cos[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  1/d * Subst[Int[Simplify[TrigExpand[a*Cos[m*ArcTan[x]]+b*Cos[n*ArcTan[x]]]]^p/(1+x^2),x],x,Tan[c+d*x]] /;
FreeQ[{a,b,c,d},x] && ILtQ[p/2,0] && IntegerQ[(m-1)/2] && IntegerQ[(n-1)/2]
```

2: $\int (a \operatorname{Sin}[m (c+d x)] + b \operatorname{Sin}[n (c+d x)])^p \, dx$ when $\frac{p-1}{2} \in \mathbb{Z}^- \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$

- **Derivation: Integration by substitution**
- **Basis: If** $\frac{p-1}{2} \in \mathbb{Z} \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$**, then**
$(a \operatorname{Sin}[m (c+d x)] + b \operatorname{Sin}[n (c+d x)])^p = -\frac{1}{d} \operatorname{Subst}\left[\frac{(a \operatorname{Sin}[m \operatorname{ArcCos}[x]] + b \operatorname{Sin}[n \operatorname{ArcCos}[x]])^p}{\sqrt{1-x^2}}, x, \operatorname{Cos}[c+d x]\right] \partial_x \operatorname{Cos}[c+d x]$
- **Basis: If** $\frac{p-1}{2} \in \mathbb{Z} \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$**, then**
$(a \operatorname{Cos}[m (c+d x)] + b \operatorname{Cos}[n (c+d x)])^p = \frac{1}{d} \operatorname{Subst}\left[\frac{(a \operatorname{Cos}[m \operatorname{ArcSin}[x]] + b \operatorname{Cos}[n \operatorname{ArcSin}[x]])^p}{\sqrt{1-x^2}}, x, \operatorname{Sin}[c+d x]\right] \partial_x \operatorname{Sin}[c+d x]$
- **Note: If** $\frac{m-1}{2} \in \mathbb{Z}$**, then** $\operatorname{Sin}[m \operatorname{ArcCos}[x]]$ **equals** $\sqrt{1-x^2}$ **times a** $\frac{m-1}{2}$ **degree polynomial in** $x^2$**.**
- **Note: If** $\frac{m-1}{2} \in \mathbb{Z}$**, then** $\operatorname{Cos}[m \operatorname{ArcSin}[x]]$ **equals** $\sqrt{1-x^2}$ **times a** $\frac{m-1}{2}$ **degree polynomial in** $x^2$**.**
- **Rule: If** $\frac{p-1}{2} \in \mathbb{Z}^- \bigwedge \frac{m-1}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$**, then**

$$\int (a \operatorname{Sin}[m (c+d x)] + b \operatorname{Sin}[n (c+d x)])^p \, dx \rightarrow -\frac{1}{d} \operatorname{Subst}\left[\int \frac{(a \operatorname{Sin}[m \operatorname{ArcCos}[x]] + b \operatorname{Sin}[n \operatorname{ArcCos}[x]])^p}{\sqrt{1-x^2}} \, dx, x, \operatorname{Cos}[c+d x]\right]$$

$$\int (a \operatorname{Cos}[m (c+d x)] + b \operatorname{Cos}[n (c+d x)])^p \, dx \rightarrow \frac{1}{d} \operatorname{Subst}\left[\int \frac{(a \operatorname{Cos}[m \operatorname{ArcSin}[x]] + b \operatorname{Cos}[n \operatorname{ArcSin}[x]])^p}{\sqrt{1-x^2}} \, dx, x, \operatorname{Sin}[c+d x]\right]$$

- **Program code:**

```
Int[(a_.*sin[m_.*(c_.+d_.*x_)]+b_.*sin[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  -1/d * Subst[Int[Simplify[TrigExpand[a*Sin[m*ArcCos[x]]+b*Sin[n*ArcCos[x]]]]^p/Sqrt[1-x^2],x],x,Cos[c+d*x]] /;
FreeQ[{a,b,c,d},x] && ILtQ[(p-1)/2,0] && IntegerQ[(m-1)/2] && IntegerQ[(n-1)/2]
```

```
Int[(a_.*cos[m_.*(c_.+d_.*x_)]+b_.*cos[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  1/d * Subst[Int[Simplify[TrigExpand[a*Cos[m*ArcSin[x]]+b*Cos[n*ArcSin[x]]]]^p/Sqrt[1-x^2],x],x,Sin[c+d*x]] /;
FreeQ[{a,b,c,d},x] && ILtQ[(p-1)/2,0] && IntegerQ[(m-1)/2] && IntegerQ[(n-1)/2]
```

**3:** $\int (a \,\text{Sin}[m\,(c+d\,x)] + b\,\text{Sin}[n\,(c+d\,x)])^p\,dx$ when $p \in \mathbb{Z}^- \bigwedge \frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n-1}{2} \in \mathbb{Z}$

- **Derivation: Integration by substitution**
- **Basis: If $m \in \mathbb{Z} \land n \in \mathbb{Z}$, then**
  $F[\text{Sin}[m\,(c+d\,x)],\,\text{Sin}[n\,(c+d\,x)]] == \frac{2}{d}\,\text{Subst}\left[\frac{F[\text{Sin}[2\,m\,\text{ArcTan}[x]],\text{Sin}[2\,n\,\text{ArcTan}[x]]]}{1+x^2},\,x,\,\text{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]\right]\,\partial_x \text{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]$
- **Basis: If $m \in \mathbb{Z} \land n \in \mathbb{Z}$, then**
  $F[\text{Cos}[m\,(c+d\,x)],\,\text{Cos}[n\,(c+d\,x)]] == -\frac{2}{d}\,\text{Subst}\left[\frac{F[\text{Cos}[2\,m\,\text{ArcCot}[x]],\text{Cos}[2\,n\,\text{ArcCot}[x]]]}{1+x^2},\,x,\,\text{Cot}\left[\frac{1}{2}\,(c+d\,x)\right]\right]\,\partial_x \text{Cot}\left[\frac{1}{2}\,(c+d\,x)\right]$
- **Note: If $m \in \mathbb{Z}$, then $\text{Sin}[2\,m\,\text{ArcTan}[x]]$ equals $x$ times a rational functions in $x^2$.**
- **Note: If $m \in \mathbb{Z}$, then $\text{Cos}[2\,m\,\text{ArcCot}[x]]$ equals a rational functions in $x^2$.**
- **Rule: If $p \in \mathbb{Z}^- \bigwedge \frac{m}{2} \in \mathbb{Z} \bigwedge \frac{n}{2} \in \mathbb{Z}$, then**

$$\int (a\,\text{Sin}[m\,(c+d\,x)] + b\,\text{Sin}[n\,(c+d\,x)])^p\,dx \longrightarrow \frac{2}{d}\,\text{Subst}\left[\int \frac{(a\,\text{Sin}[2\,m\,\text{ArcTan}[x]] + b\,\text{Sin}[2\,n\,\text{ArcTan}[x]])^p}{1+x^2}\,dx,\,x,\,\text{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]\right]$$

- **Program code:**

```
Int[(a_.*sin[m_.*(c_.+d_.*x_)]+b_.*sin[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  2/d * Subst[Int[Simplify[TrigExpand[a*Sin[2*m*ArcTan[x]]+b*Sin[2*n*ArcTan[x]]]]^p/(1+x^2),x],x,Tan[1/2*(c+d*x)]] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && IntegerQ[m/2] && IntegerQ[(n-1)/2]
```

```
Int[(a_.*cos[m_.*(c_.+d_.*x_)]+b_.*cos[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  -2/d * Subst[Int[Simplify[TrigExpand[a*Cos[2*m*ArcCot[x]]+b*Cos[2*n*ArcCot[x]]]]^p/(1+x^2),x],x,Cot[1/2*(c+d*x)]] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && IntegerQ[m/2] && IntegerQ[(n-1)/2]
```

# Rules for integrands of the form $(a\,\text{Sin}[m\,(c+d\,x)]+b\,\text{Cos}[n\,(c+d\,x)])^p$

**1:** $\int (a\,\text{Sin}[m\,(c+d\,x)]+b\,\text{Cos}[n\,(c+d\,x)])^p\,dx$ when $p\in\mathbb{Z}^-\bigwedge\frac{m}{2}\in\mathbb{Z}\bigwedge\frac{n}{2}\in\mathbb{Z}$

- **Derivation: Integration by substitution**
- **Basis: If** $\frac{m}{2}\in\mathbb{Z}\bigwedge\frac{n}{2}\in\mathbb{Z}$**, then**
  $F[\text{Sin}[m\,(c+d\,x)],\,\text{Cos}[n\,(c+d\,x)]] = \frac{1}{d}\,\text{Subst}\left[\frac{F[\text{Sin}[m\,\text{ArcTan}[x]],\text{Cos}[n\,\text{ArcTan}[x]]]}{1+x^2},\,x,\,\text{Tan}[c+d\,x]\right]\partial_x\text{Tan}[c+d\,x]$
- **Note: If** $\frac{m}{2}\in\mathbb{Z}$**, then** $\text{Sin}[m\,\text{ArcTan}[x]]$ **equals** $x$ **times a rational functions in** $x^2$**.**
- **Note: If** $\frac{m}{2}\in\mathbb{Z}$**, then** $\text{Cos}[m\,\text{ArcTan}[x]]$ **equals a rational functions in** $x^2$**.**
- **Rule: If** $p\in\mathbb{Z}^-\bigwedge\frac{m}{2}\in\mathbb{Z}\bigwedge\frac{n}{2}\in\mathbb{Z}$**, then**

$$\int (a\,\text{Sin}[m\,(c+d\,x)]+b\,\text{Cos}[n\,(c+d\,x)])^p\,dx \longrightarrow \frac{1}{d}\,\text{Subst}\left[\int\frac{(a\,\text{Sin}[m\,\text{ArcTan}[x]]+b\,\text{Cos}[n\,\text{ArcTan}[x]])^p}{1+x^2}\,dx,\,x,\,\text{Tan}[c+d\,x]\right]$$

- **Program code:**

```
Int[(a_.*sin[m_.*(c_.+d_.*x_)]+b_.*cos[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  1/d * Subst[Int[Simplify[TrigExpand[a*Sin[m*ArcTan[x]]+b*Cos[n*ArcTan[x]]]]^p/(1+x^2),x],x,Tan[c+d*x]] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && IntegerQ[m/2] && IntegerQ[n/2]
```

2: $\int (a \operatorname{Sin}[m\,(c+d\,x)]+b \operatorname{Cos}[n\,(c+d\,x)])^p\,dx$ when $\frac{p-1}{2}\in\mathbb{Z}^- \bigwedge \frac{m}{2}\in\mathbb{Z} \bigwedge \frac{n-1}{2}\in\mathbb{Z}$

- **Derivation: Integration by substitution**
- **Basis: If $\frac{p-1}{2}\in\mathbb{Z} \bigwedge \frac{m}{2}\in\mathbb{Z} \bigwedge \frac{n-1}{2}\in\mathbb{Z}$, then**
  $(a \operatorname{Sin}[m\,(c+d\,x)]+b \operatorname{Cos}[n\,(c+d\,x)])^p = \frac{1}{d}\operatorname{Subst}\left[\frac{(a \operatorname{Sin}[m \operatorname{ArcSin}[x]]+b \operatorname{Cos}[n \operatorname{ArcSin}[x]])^p}{\sqrt{1-x^2}},\,x,\,\operatorname{Sin}[c+d\,x]\right]\partial_x \operatorname{Sin}[c+d\,x]$
- **Note: If $\frac{m}{2}\in\mathbb{Z}$, then $\operatorname{Sin}[m \operatorname{ArcSin}[x]]$ equals $x\sqrt{1-x^2}$ times a $\frac{m-2}{2}$ degree polynomial in $x^2$.**
- **Note: If $\frac{m-1}{2}\in\mathbb{Z}$, then $\operatorname{Cos}[m \operatorname{ArcSin}[x]]$ equals $\sqrt{1-x^2}$ times a $\frac{m-1}{2}$ degree polynomial in $x^2$.**
- **Rule: If $\frac{p-1}{2}\in\mathbb{Z}^- \bigwedge \frac{m}{2}\in\mathbb{Z} \bigwedge \frac{n-1}{2}\in\mathbb{Z}$, then**

$$\int (a \operatorname{Sin}[m\,(c+d\,x)]+b \operatorname{Cos}[n\,(c+d\,x)])^p\,dx \longrightarrow \frac{1}{d}\operatorname{Subst}\left[\int \frac{(a \operatorname{Sin}[m \operatorname{ArcSin}[x]]+b \operatorname{Cos}[n \operatorname{ArcSin}[x]])^p}{\sqrt{1-x^2}}\,dx,\,x,\,\operatorname{Sin}[c+d\,x]\right]$$

- **Program code:**

```
Int[(a_.*sin[m_.*(c_.+d_.*x_)]+b_.*cos[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  1/d * Subst[Int[Simplify[TrigExpand[a*Sin[m*ArcSin[x]]+b*Cos[n*ArcSin[x]]]]^p/Sqrt[1-x^2],x],x,Sin[c+d*x]] /;
FreeQ[{a,b,c,d},x] && ILtQ[(p-1)/2,0] && IntegerQ[m/2] && IntegerQ[(n-1)/2]
```

**3:** $\int (a\,\mathrm{Sin}[m\,(c+d\,x)]+b\,\mathrm{Cos}[n\,(c+d\,x)])^p\,dx$ when $p\in\mathbb{Z}^-\wedge m\in\mathbb{Z}\wedge n\in\mathbb{Z}$

- **Derivation: Integration by substitution**
- **Basis: If $m\in\mathbb{Z}\wedge n\in\mathbb{Z}$, then**
$F[\mathrm{Sin}[m\,(c+d\,x)],\,\mathrm{Cos}[n\,(c+d\,x)]]=\frac{2}{d}\,\mathrm{Subst}\left[\frac{F[\mathrm{Sin}[2\,m\,\mathrm{ArcTan}[x]],\mathrm{Cos}[2\,n\,\mathrm{ArcTan}[x]]]}{1+x^2},\,x,\,\mathrm{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]\right]\partial_x\mathrm{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]$
- **Note: If $m\in\mathbb{Z}$, then $\mathrm{Sin}[2\,m\,\mathrm{ArcTan}[x]]$ equals $x$ times a rational functions in $x^2$.**
- **Note: If $m\in\mathbb{Z}$, then $\mathrm{Cos}[2\,m\,\mathrm{ArcTan}[x]]$ equals a rational functions in $x^2$.**
- **Rule: If $p\in\mathbb{Z}^-\wedge m\in\mathbb{Z}\wedge n\in\mathbb{Z}$, then**
$$\int (a\,\mathrm{Sin}[m\,(c+d\,x)]+b\,\mathrm{Cos}[n\,(c+d\,x)])^p\,dx\to\frac{2}{d}\,\mathrm{Subst}\left[\int\frac{(a\,\mathrm{Sin}[2\,m\,\mathrm{ArcTan}[x]]+b\,\mathrm{Cos}[2\,n\,\mathrm{ArcTan}[x]])^p}{1+x^2}\,dx,\,x,\,\mathrm{Tan}\left[\frac{1}{2}\,(c+d\,x)\right]\right]$$
- **Program code:**

```
Int[(a_.*sin[m_.*(c_.+d_.*x_)]+b_.*cos[n_.*(c_.+d_.*x_)])^p_,x_Symbol] :=
  2/d * Subst[Int[Simplify[TrigExpand[a*Sin[2*m*ArcTan[x]]+b*Cos[2*n*ArcTan[x]]]]^p/(1+x^2),x],x,Tan[1/2*(c+d*x)]] /;
FreeQ[{a,b,c,d},x] && ILtQ[p,0] && IntegerQ[m] && IntegerQ[n]
```